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The Impact of Inclination Angle and Thermal Load on Flow Patterns in a Bilayer System Taking into Account the Mass Transfer

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Abstract. Bilayer convective flows of liquid and gas-vapor mixture in an inclined channel are modeled taking into account the heat and mass transfer at the thermocapillary interface. Mathematical modeling is based on the exact solutions of special type of the Navier-Stokes equations in the Oberbeck-Boussinesq approximation, with consideration of the Soret and Dufour effects in the gas-vapor layer. Inclined or horizontal position of the channel and direction of the boundary thermal load determine a form of the exact solution and an algorithm of its construction. Examples of the velocity profiles, temperature and vapor concentration distributions in the «ethanol — nitrogen» system are demonstrated. Results of comparative analysis of the two-layer flows in the system positioned horizontally and by small inclination from the horizontal level are presented. The influence of the thermal load intensity on the flow and mass transfer characteristics is studied.

Keywords: exact solution, bilayer flow, thermocapillary interface, convection, mass transfer, inclined channel.

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Introduction

Convective flows of liquids are often observed in the natural and technological processes. One of the main mechanisms determining the character of such flows is the heat and mass transfer. In addition, the reciprocal effects of the diffusive thermal conductivity and thermodiffusion influence on the fluid flows induced both by temperature and concentration inhomogeneities [1].

The interest in construction of the exact solutions describing convective flows with interfaces is provoked due to a possibility of promt analyzing the influence of various parameters of the problem on the processes. This allows one to identify the dominant mechanisms influencing the flow topology, to improve experimental techniques, and to predict the results of experimental study of dynamics of the liquids and co-current gases [2]. Due to the fact that the models of the

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convective heat and mass transfer are generally nonlinear, construction of their exact solutions can be a rather interesting problem from the mathematical point of view.

Nowadays, there are quite many works devoted to the construction of exact solutions describing the flows of thermally conductive fluids. In [3, 4] special solutions modeling one-directional flows are proposed. The group nature of the solutions obtained in [4] is revealed in [5]. The article [6] is devoted to generalization of the proposed solutions for the non-stationary case in a plane layer and a rotating tube. The example of exact solution describing flows of an one-directional binary fluid is proposed in the work [7].

Due to nonlinearity of the system of the Oberbeck-Bussinesq equations, the introducing additional effects or parameters complicates the process of problem solving. Construction of exact solutions in such case is connected with generalization of known solutions. The work [8] is devoted to construction of exact solutions describing the two-layer flows in a «liquid-liquid» system taking into account the mass transfer in a horizontal channel. The process of liquid vaporization into the gas-vapor layer under closed flow conditions is considered in [9]. Modeling of the bilayer flows with respect to a given gas flow rate in the upper layer is carried out in [10]. In [11] paper, in addition to the diffusive thermal conductivity effect in the gas-vapor layer, the thermodiffusion process is also taken into account. The stability issues of the presented exact solutions were studied in [12, 13]. Comparison of analytical results concerning the liquid evaporation into the gas-vapor layer with experimental data is presented in [11].

An additional parameter complicating the construction of exact solutions of the Navier-Stokes equations in the Oberbeck–Boussinesq approximation is also geometry of a flow domain. Mathematical modeling in the horizontal, vertical and inclined layers is carried out in [14]. In [15], a variant of such solution is proposed for the problem of fluid flow in an inclined channel with moving solid boundaries on which a longitudinal temperature gradient is given. Bilayer «liquid-gas» systems with consideration of evaporation at the thermocapillary interface and given inclination angle of the channel are considered in [16]. Here, the condition of total vapor absorption was assumed to be satisfied on the upper channel wall.

This paper presents exact solutions of special type of the Navier–Stokes equations in the Boussinesq approximation describing flows in an inclined channel filled by liquid and gas-vapor mixture. The thermocapillary interface is assumed to be non-deformable (see [12, 17]). In the upper layer of the system, the Soret and Dufour effects are taken into account and the gas flow rate is given. As a condition for the vapor concentration function on the upper channel wall the condition of zero vapor flux is chosen. The obtained solution is compared with the one presented in [11, 18] for the horizontal layer. The impact of physical and geometrical parameters of the problem on the flow patterns is studied.

1. Problem statement of convection in an inclined channel in the case of a non-deformable interface

1.1. Governing equations

The stationary flow in the «liquid–gas» system in an inclined layer with solid impermeable walls is studied. A viscous incompressible liquid and a bicomponent mixture of gas and vapor occupy an infinite channel. The thicknesses of the layers are fixed and equal to l and h, respectively (see Fig. 1). The Cartesian coordinate system is positioned such that the interface, which remains non-deformable, is determined by the equation $y = 0$. The mass force vector $\mathbf{g}$

is directed at the angle φ with respect to the substrate ($\mathbf{g} = (g\cos\varphi, -g\sin\varphi)$). Note, that the inclination angle from the horizontal plane is defined as $(\pi/2 - \varphi)$. Vapor is a passive impurity in the gas phase, i.e. it does not affect the properties of the gas.

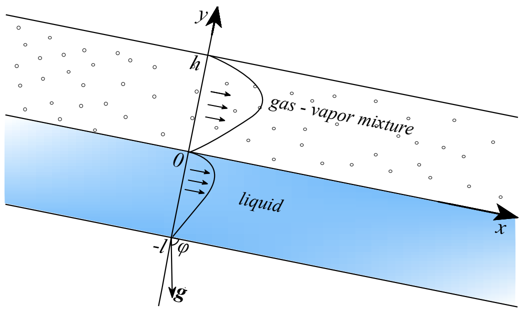

Fig. 1. Flow area geometry

Mathematical modeling of the liquid and gas-vapor mixture flows is carried out by using the Navier–Stokes equations in the Oberbeck–Boussinesq approximation. The vapor concentration function Φ satisfies the diffusion equation. The Soret and Dufour effects are taken into account in the upper layer.

The exact solutions of the governing equations will be constructed in the special Ostroumov-Birikh form [3, 4] (see [8]):

$$u_i = u_i(y), \quad v_i = 0, \quad T_i = Ax + \vartheta_i(y), \quad \Phi = -Bx + \psi(y), \quad p'_i = p'_i(x, y), \tag{1}$$

where u_i and v_i are projections of the velocity vector on the Cartesian coordinate system axes, p'_i is modified pressure or deviation from the hydrostatic pressure ($p'_i = p_i - \rho_i\,\mathbf{g}\cdot\mathbf{x}$, $\mathbf{x} = (x, y)$, p_i — pressure, ρ_i — density), T_i — temperature, Φ — vapor concentration in the gas, A, B — parameters defining longitudinal gradients of temperature and vapor concentration, ϑ_i, ψ — terms included in the definition of the functions T_i and Φ and depending only on the longitudinal coordinate. Hereinafter, i defines the number of the system layer: functions and parameters describing the fluid flow are marked by the index $i = 1$, gas-vapor mixture by $i = 2$. It should be noted that the Birikh solution, which is a special case of the solution (1) received experimental and numerical confirmation (see, for example, [19]). In addition, the results obtained using the solution (1) are confirmed by experimental data in [2, 11].

The system of equations describing flows in the lower layer filled with one-component fluid is written as follows according to (1):

$$\frac{1}{\rho_1}p'_{1\,x} = \nu_1 u_{1\,yy} - g\cos\varphi\beta_1 T_1, \qquad \frac{1}{\rho_1}p'_{1\,y} = g\sin\varphi\beta_1 T_1, \qquad u_1 T_{1x} = \chi_1 T_{1\,yy}. \tag{2}$$

In the upper layer containing gas and vapor mixture, the system of governing equations should be supplemented by the diffusion equation for the vapor concentration function:

$$\begin{gathered}\frac{1}{\rho_2}p'_{2\,x} = \nu_2 u_{2\,yy} - g\cos\varphi(\beta_2 T_2 + \gamma\Phi), \qquad \frac{1}{\rho_2}p'_{2\,y} = g\sin\varphi(\beta_2 T_2 + \gamma\Phi), \\ u_2 T_{2x} = \chi_2 T_{2\,yy} + \chi_2\delta\Phi_{yy}, \qquad u_2\Phi_x = D\Phi_{yy} + \alpha D T_{2\,yy}.\end{gathered} \tag{3}$$

In equations (2), (3) the following notations are used: ν_i — kinematic viscosity coefficients, β_i — thermal expansion coefficients, χ_i — heat diffusivity coefficients, γ — concentration density coefficient, D — vapor diffusion coefficient in the gas, coefficients α and δ characterize the Soret and Dufour effects, respectively.

1.2. Boundary conditions

Let us formulate the conditions on the boundaries of the system. The no-slip condition for velocity should be fulfilled on the solid walls of the channel:

$$u_1|_{y=-l} = 0, \quad u_2|_{y=h} = 0. \tag{4}$$

The linear thermal regime is set on the channel walls:

$$T_1|_{y=-l} = Ax + \vartheta^-, \quad T_2|_{y=h} = Ax + \vartheta^+. \tag{5}$$

Here ϑ^- and ϑ^+ are some fixed constants.

At the boundary $y = h$, the vapor concentration function Φ satisfies the condition of zero vapor flux:

$$(\Phi_y + \alpha T_{2\,y})|_{y=h} = 0. \tag{6}$$

At the thermocapillary interface $y = 0$, the conditions of continuity of the velocity and temperature functions are satisfied:

$$u_1|_{y=0} = u_2|_{y=0}, \quad T_1|_{y=0} = T_2|_{y=0}. \tag{7}$$

The kinematic condition ($v_1 = 0$ and $v_2 = 0$) is satisfied automatically by virtue of the type of the exact solution (1). The projection of the dynamic condition onto the tangent vector to the interface is written as follows:

$$\rho_1\nu_1 u_{1\,y} = \rho_2\nu_2 u_{2\,y} + \sigma_T T_{1\,x}|_{y=0}, \tag{8}$$

where σ_T is the temperature coefficient of surface tension σ. The linear dependence of surface tension on temperature is assumed: $\sigma = \sigma_0 + \sigma_T(T - T_0)$, σ_0 — surface tension at some reference temperature T_0, $\sigma_T = const$, $\sigma_T < 0$. The dynamic condition expresses the tangential stress balance at the interface.

The heat flux balance, taking into account the diffusion mass flux of the vaporizing liquid at the interface M and the diffusive thermal conductivity effect, is [10, 11, 20]:

$$\kappa_1 T_{1\,y} - \kappa_2 T_{2\,y} - \delta\kappa_2\Phi_y|_{y=0} = -LM, \quad M = -D\rho_2(\Phi_y|_{y=0} + \alpha T_{2\,y}|_{y=0}). \tag{9}$$

Here the following notations are accepted: L is the latent heat of evaporation, M is the mass velocity of liquid evaporating from a unit surface area per unit time ($M = const$), κ_1 and κ_2 are thermal conductivity coefficients of liquid and gas-vapor mixture, respectively.

The saturated vapor concentration at the interface is determined according to the following relation (see [11]):

$$\Phi|_{y=0} = \Phi_*(1 + \varepsilon(T_2|_{y=0} - T_0)), \tag{10}$$

where ε is a parameter depending on the characteristic temperature and physical and chemical properties of the medium, Φ_* is the concentration of saturated vapor at $T_2 = T_0$.

The problem is solved under the condition of given liquid flow rate Q_1 and gas flow rate Q_2.

$$Q_1 = \int_{-l}^{0} \rho_1 u_1(y)dy, \quad Q_2 = \int_{0}^{h} \rho_2 u_2(y)dy. \tag{11}$$

2. Constructing the exact solution of a special type

The exact solution of the problem is constructed by substituting formulas (1) into the differential equations (2), (3). The functions p_i' are eliminated by cross differentiation of the first two relations in (2), (3). Performing further subsequent differentiation of the obtained expressions on the variable y we have $u_{1\,y}^{(4)} + \lambda_1 u_1 = 0$ and $u_{2\,y}^{(4)} + \lambda_2 u_2 = 0$. The solutions of obtained equations are the functions u_1 and u_2, which determine the longitudinal velocities in each of the layers of the system. The coefficients λ_1 and λ_2 depend on the geometric and physical and chemical parameters of the working media and have the following form: $\lambda_1 = -Ag\cos\varphi\beta_1/(\chi_1\nu_1)$, $\lambda_2 = -Ag\cos\varphi E$, where $E = [D(\beta_2-\alpha\gamma)-\chi_2 C_*\varepsilon(\delta\beta_2-\gamma)]/[\chi_2\nu_2 D(1-\alpha\delta)]$. The function ϑ_1 is found by integration from the heat transfer equation (see the third expression in (2)). Similarly, the functions ϑ_2 and ψ are recovered from the heat transfer and diffusion equations (see the third and fourth expressions in (3)). The inequalities $\lambda_1 < 0$ and $\lambda_2 > 0$ when $A > 0$, and $\lambda_1 > 0$, $\lambda_2 < 0$ when $A < 0$ are true for systems of the «ethanol — nitrogen» type. In the first case, the desired functions are represented as the following analytic expressions [22] (see also [21]):

$$
\begin{aligned}
&u_1 = C_1\sin(k_1y) + C_2\cos(k_1y) + C_3\,\mathrm{sh}(k_1y) + C_4\,\mathrm{ch}(k_1y),\\
&u_2 = \overline{C}_1\sin(m_1y)\,\mathrm{sh}(m_1y) + \overline{C}_2\cos(m_1y)\,\mathrm{sh}(m_1y) + \overline{C}_3\sin(m_1y)\,\mathrm{ch}(m_1y)+\\
&\qquad\qquad + \overline{C}_4\cos(m_1y)\,\mathrm{ch}(m_1y),\\
&T_1(x,y) = Ax + \frac{F_1}{k_1^2}\Big(-C_1\sin(k_1y) - C_2\cos(k_1y) + C_3\,\mathrm{sh}(k_1y) + C_4\,\mathrm{ch}(k_1y)\Big) + C_5y + C_6,\\
&T_2(x,y) = Ax + \frac{F_2}{2m_1^2}\Big(-\overline{C}_1\cos(m_1y)\,\mathrm{ch}(m_1y) + \overline{C}_2\sin(m_1y)\,\mathrm{ch}(m_1y)-\\
&\qquad\qquad -\overline{C}_3\cos(m_1y)\,\mathrm{sh}(m_1y) + \overline{C}_4\sin(m_1y)\,\mathrm{sh}(m_1y)\Big) + \overline{C}_5y + \overline{C}_6,\\
&\Phi(x,y) = -Bx + \frac{G}{2m_1^2}\Big(-\overline{C}_1\cos(m_1y)\,\mathrm{ch}(m_1y) + \overline{C}_2\sin(m_1y)\,\mathrm{ch}(m_1y)-\\
&\qquad\qquad -\overline{C}_3\cos(m_1y)\,\mathrm{sh}(m_1y) + \overline{C}_4\sin(m_1y)\,\mathrm{sh}(m_1y)\Big) + \overline{C}_7y + \overline{C}_8.
\end{aligned}
\tag{12}
$$

At negative value of the parameter A determining the longitudinal temperature gradient, the functions describing the flow patterns take the form presented in details in [22]:

$$
\begin{aligned}
&u_1(y) = C_1\sin(k_2y)\,\mathrm{sh}(k_2y) + C_2\cos(k_2y)\,\mathrm{sh}(k_2y)+\\
&\qquad\qquad + C_3\sin(k_2y)\,\mathrm{ch}(k_2y) + C_4\cos(k_2y)\,\mathrm{ch}(k_2y),\\
&u_2(y) = \overline{C}_1\sin(m_2y) + \overline{C}_2\cos(m_2y) + \overline{C}_3\,\mathrm{sh}(m_2y) + \overline{C}_4\,\mathrm{ch}(m_2y),\\
&T_1(x,y) = Ax + \frac{F_1}{2k_2^2}\Big(-C_1\cos(k_2y)\,\mathrm{ch}(k_2y) + C_2\sin(k_2y)\,\mathrm{ch}(k_2y)-\\
&\qquad\qquad - C_3\cos(k_2y)\,\mathrm{sh}(k_2y) + C_4\sin(k_2y)\,\mathrm{sh}(k_2y)\Big) + C_5y + C_6,\\
&T_2(x,y) = Ax + \frac{F_2}{m_2^2}\Big(-\overline{C}_1\sin(m_2y) - \overline{C}_2\cos(m_2y) + \overline{C}_3\,\mathrm{sh}(m_2y) + \overline{C}_4\,\mathrm{ch}(m_2y)\Big)+\\
&\qquad\qquad + \overline{C}_5y + \overline{C}_6,\\
&\Phi(x,y) = -Bx + \frac{G}{m_2^2}\Big(-\overline{C}_1\sin(m_2y) - \overline{C}_2\cos(m_2y) + \overline{C}_3\,\mathrm{sh}(m_2y) + \overline{C}_4\,\mathrm{ch}(m_2y)\Big)+\\
&\qquad\qquad + \overline{C}_7y + \overline{C}_8.
\end{aligned}
\tag{13}
$$

The coefficients k_s, m_s, F_1, F_2, G are calculated through the problem parameters as follows: $k_1 = \sqrt[4]{Ag\cos\varphi\beta_1/(\chi_1\nu_1)}$, $k_2 = \sqrt[4]{-Ag\cos\varphi\beta_1/(4\chi_1\nu_1)}$, $m_1 = \sqrt[4]{Ag\cos\varphi E/4}$, $m_2 = \sqrt[4]{Ag\cos\varphi E}$, $F_1 = A/\chi_1$, $F_2 = A(D-\delta\chi_2 C_*\varepsilon)/[\chi_2 D(1-\alpha\delta)]$, $G = A(\alpha D-\chi_2 C_*\varepsilon)/[\chi_2 D(\alpha\delta-1)]$. The index s defines the solutions for $A > 0$ ($s = 1$) and $A < 0$ ($s = 2$). The coefficients C_i and $\overline{C}_i$ ($i = 1, \dots, 8$) are different integration constants for each of the systems (12), (13). These constants satisfies the systems of algebraic equations resulting from the conditions on the solid walls and interface (4)–(10) and the expressions (11) that determine the gas and liquid flow rates (see [22]). The resulting systems is nonclosed. For closure, the constants C_6 and $\overline{C}_6$, which are included as free terms in the temperature function, can be assumed, for instance, to be zero. The pressure functions p'_i are recovered by their partial derivatives from the first two equations of the systems (2), (3). Note, that the expression defining the saturated vapor concentration at the interface (10) dictates the condition of compatibility of the problem parameters defining the longitudinal gradients of temperature and vapor concentration: $B = -\Phi_*\varepsilon A$.

Let us consider a special case when the angle value $\varphi = 90°$, i.e. the channel position becomes horizontal (see Fig. 1). Then the components of the mass force vector take the form (0, -g). In [10, 11, 18], exact solutions of special type are proposed, where the temperature and vapor concentration functions are written as follows: $T_i = (A + \bar{A}y)x + \vartheta_i(y)$, $\Phi = -(B + \bar{B}y)x + \psi(y)$. In this paper, the solutions given in the [10, 11, 18] taking into account the formulas (1) with $\bar{A} = \bar{B} = 0$ are written in the following simplified form:

$$u_1 = \frac{g\beta_1}{\nu_1}\frac{y^3}{6}A + \frac{y^2}{2}c_1 + yc_2 + c_3, \quad u_2 = \frac{g}{\nu_2}\frac{y^3}{6}(\beta_2 A + \gamma B) + \bar{c}_1\frac{y^2}{2} + \bar{c}_2 y + \bar{c}_3,$$

$$T_1 = Ax + \frac{y^5}{120}\frac{g\beta_1(A)^2}{\nu_1\chi_1} + \frac{y^4}{24}\frac{c_1 A}{\chi_1} + \frac{y^3}{6}\frac{c_2 A}{\chi_1} + \frac{y^2}{2}\frac{c_3 A}{\chi_1} + yc_4 + c_5,$$

$$T_2 = Ax + +\frac{y^5}{120}\frac{g}{\nu_2}\Big(\frac{A}{\chi_2} - \delta\frac{B}{D}\Big)(\beta_2 A + \gamma B) + \frac{y^4}{24}\Big(\frac{A}{\chi_2} - \delta\frac{B}{D}\Big)\bar{c}_1 +$$
$$+\frac{y^3}{6}\Big(\frac{A}{\chi_2} - \delta\frac{B}{D}\Big)\bar{c}_2 + \frac{y^2}{2}\Big(\frac{A}{\chi_2} - \delta\frac{B}{D}\Big)\bar{c}_3 + y\bar{c}_4 + \bar{c}_5, \tag{14}$$

$$\Phi = Bx + \frac{y^5}{120}\frac{g}{\nu_2}\frac{B}{D}(\beta_2 A + \gamma B) + \frac{y^4}{24}\frac{B}{D}\bar{c}_1 + \frac{y^3}{6}\frac{B}{D}\bar{c}_2 + \frac{y^2}{2}\frac{B}{D}\bar{c}_3 + y\bar{c}_6 + \bar{c}_7,$$

where c_i, $\bar{c}_i$ are integration constants determined by conditions at the boundaries of the system. The first relation (7) entails the equality $c_3 = \bar{c}_3$. The relationship between the constants c_2 and $\bar{c}_2$ is determined from the dynamic condition (8): $c_2 = (\rho_2\nu_2)/(\rho_1\nu_1) + (\sigma_T A)/(\rho_1\nu_1)$. The unknowns c_1, $\bar{c}_1$, $\bar{c}_2$, $\bar{c}_3$ are found using the system of linear algebraic equations that is a consequence of the formulas (4), (11). The equality of constants c_5, $\bar{c}_5$ follows from the second relation in formula (7). The expression $\bar{c}_7 = \Phi_* + \Phi_*\varepsilon(\bar{c}_5 - T_0)$ is obtained using the condition (10). The mass of liquid evaporating from the interface is calculated using the relation $M = -D\rho_2(\bar{c}_6 + \alpha\bar{c}_4)$ (see mass balance equation (9)). The constant c_4 is determined from the condition (9) as follows: $c_4 = (LD\rho_2/\kappa_1 + \delta\kappa_2)\bar{c}_6 + (LD\rho_2\alpha/\kappa_1 + \kappa_2/\kappa_1)\bar{c}_4$. The unknowns $\bar{c}_4$, $\bar{c}_5$, $\bar{c}_6$ are found using the system of linear algebraic equations that is a consequence of conditions (5) and (6).

3. Examples of flows

We consider ethanol as the liquid filling the lower layer and nitrogen as the gas. The chemical parameters of the working media are given in the order {ethanol, nitrogen} (or only ethanol) according to [23]: $\rho = \{7.89 \cdot 10^2,\ 1.2\}$ kg/m^3; $\nu = \{2 \cdot 10^{-6},\ 0.15 \cdot 10^{-4}\}$ m^2/s; $\beta = \{1.079 \cdot 10^{-3},\ 3.67 \cdot 10^{-3}\}$ K^{-1}; $\chi = \{8.9 \cdot 10^{-8},\ 0.3 \cdot 10^{-4}\}$ m^2/s; $\kappa = \{0.1705,\ 0.02717\}$ W/(m·K); $\sigma_T = -0.8 \cdot 10^{-2}$ N/(m·K); $D = 1.02 \cdot 10^{-4}$ m^2/s; $L = 217$ W·s/kg; $\Phi_* = 0.1$ (corresponding to the equilibrium temperature $T_0 = 293.15$ K); $\gamma = -0.62$; $\varepsilon_* = 0.06$ K^{-1}. The thicknesses of liquid and gas-vapor layers are assumed to be 5 mm, the value of gas flow rate in the upper layer of the system is $3.6 \cdot 10^{-5}$ kg/(m·s). The Soret and Dufour coefficients are assumed to be 10^{-4} K^{-1} and 10^{-4} K, respectively. The value Q_1, determining the liquid flow rate in the lower layer, is assumed equal to zero. This corresponds to the condition of the closed flow in the lower layer, which is physically correct for small values of the inclination angle $(\pi/2 - \varphi)$ relative to the horizontal position of the channel.

3.1. The impact of channel inclination angle on the flow patterns

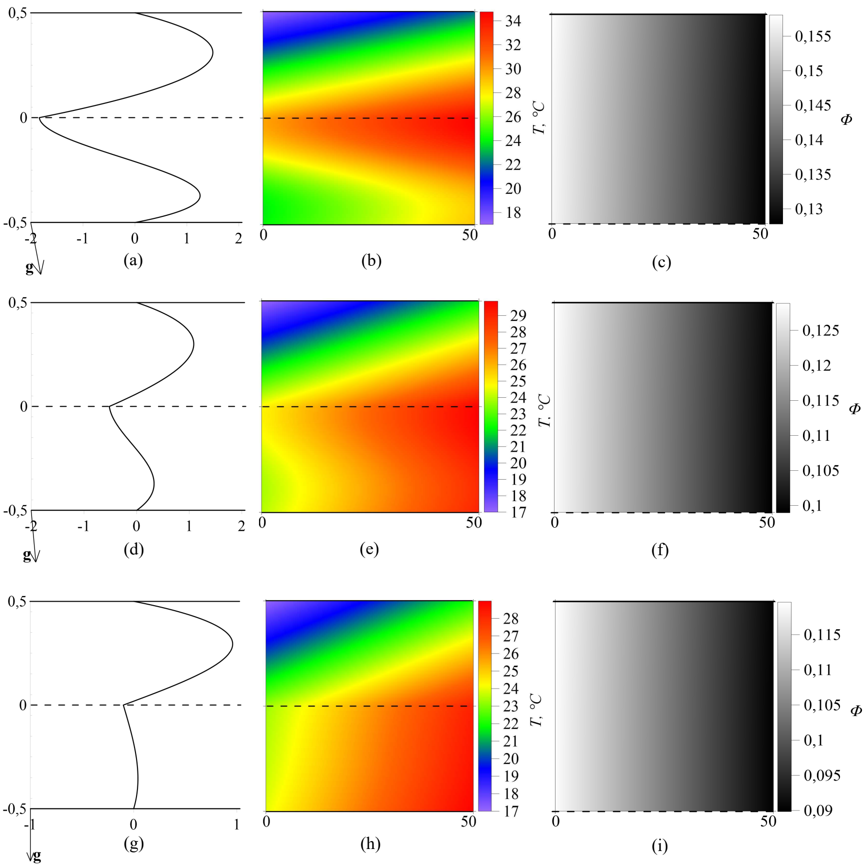

Fig. 2. Velocity profiles (a, d, g), temperature (b, e, h) and vapor concentration (c) distributions in the system: $A = 10$ K/m, $\vartheta^- = 24$ $°C$, $\vartheta^+ = 17$ $°C$, $\alpha = 10^{-4}$ K^{-1}, $\delta = 10^{-4}$ K, $g = 9.81$ m/s^2: (a,b,c) — $\varphi = 80°$ $M = 2.124 \cdot 10^{-6}$ kg/(m^2·c), (d,e,f) — $\varphi = 85°$ $M = 2.124 \cdot 10^{-6}$ kg/(m^2·c), (g,h) — $\varphi = 90°$ $M = 2.124 \cdot 10^{-6}$ kg/(m^2·c)

Let us consider the impact of angle φ on the character of flow in the system, as well as on the intensity of the liquid evaporation process into the gas-vapor layer under conditions of normal gravity. The profiles of longitudinal velocity (a, d), temperature distribution (b, e), and vapor concentration (c, f) are presented in Fig. 2 for the cases when solutions of the form (12) were used as a calculation model. For the case of horizontal layer the flow characteristics are based on formulas (14) (g, h, k). As the value of angle φ increases from 80° to 90°, which corresponds to a decrease in the channel inclination angle with respect to the horizontal plane, the intensity of the return flow near the interface decreases. The temperature distributions change both qualitatively and quantitatively. In the case when the angle φ is 80°, the formation of a thermocline near the interface is observed. In the case of decreasing $(\pi/2 - \varphi)$ angle, the zone of the highest temperature moves to the lower wall of the channel, which is caused by its additional heating ($\vartheta^- = 24\ °C$). The vapor concentration distribution at changing values of the parameter φ qualitatively remains constant, but there are some quantitative changes. As φ increases, the values of the function Φ decrease. Note, that at the same time the liquid evaporation intensity does not depend on the channel inclination angle.

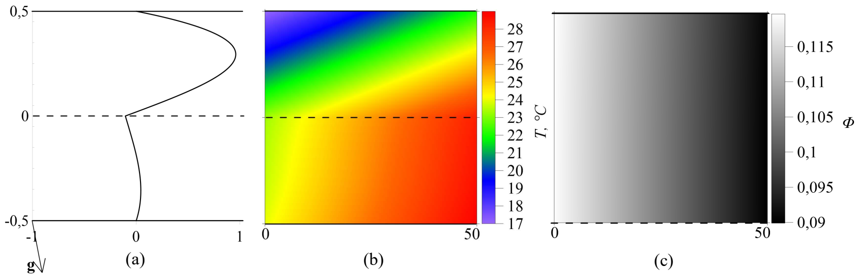

Fig. 3. Velocity profile (a), temperature (b) and vapor concentration (c) distributions in the system: $A = 10$ K/m, $\vartheta^- = 24\ °C$, $\vartheta^+ = 17\ °C$, $g = 9.81 \cdot 10^{-2}$ m/s^2: (a,b,c) – $\varphi = 80°$ $M = 2.124 \cdot 10^{-7}$ kg/(m^2·c)

There is an extremely weak dependence of the functions characterizing liquid and gas flows on the channel inclination angle under microgravity conditions. The flow patterns both qualitatively and quantitatively remain close to those obtained using the formulas (14) (see Fig. 3 and Fig. 2 (g, h, k)) when velocity profiles and temperature and vapor concentration distributions are plotted using the exact solutions (12) describing flows in the inclined layer. However, the mass flow rate of evaporation also decreases in the case of decreasing gravity.

3.2. The impact of thermal load on the flow patterns

Figs. 4 and 5 show the results illustrating the impact of the longitudinal temperature gradient on the velocity profiles, temperature and vapor concentration distributions, as well as the processes of liquid evaporation and condensation in an inclined channel. The angle φ here is 70°. Let us consider the case when $A > 0$ (Fig. 4). The growth of the parameter A value has only a quantitative effect on the longitudinal velocity in the system, while the qualitative character of the flow remains unchanged (Fig. 4 (a, d, g)). The temperature distribution changes significantly both qualitatively and quantitatively. In the case of small values of the longitudinal gradient A, the highest temperature values are observed at the lower wall of the channel, which is a consequence of its heating. As the parameter A increases, the value $\vartheta^- = 27\ °C$ has less influence on the character of temperature distributions; the highest values of the temperature functions are near the interface. This effect is accompanied by a more intensive evaporation of the lower liquid and by the increasing of values of the vapor concentration function. The character of distribution of the Φ function remains unchanged.

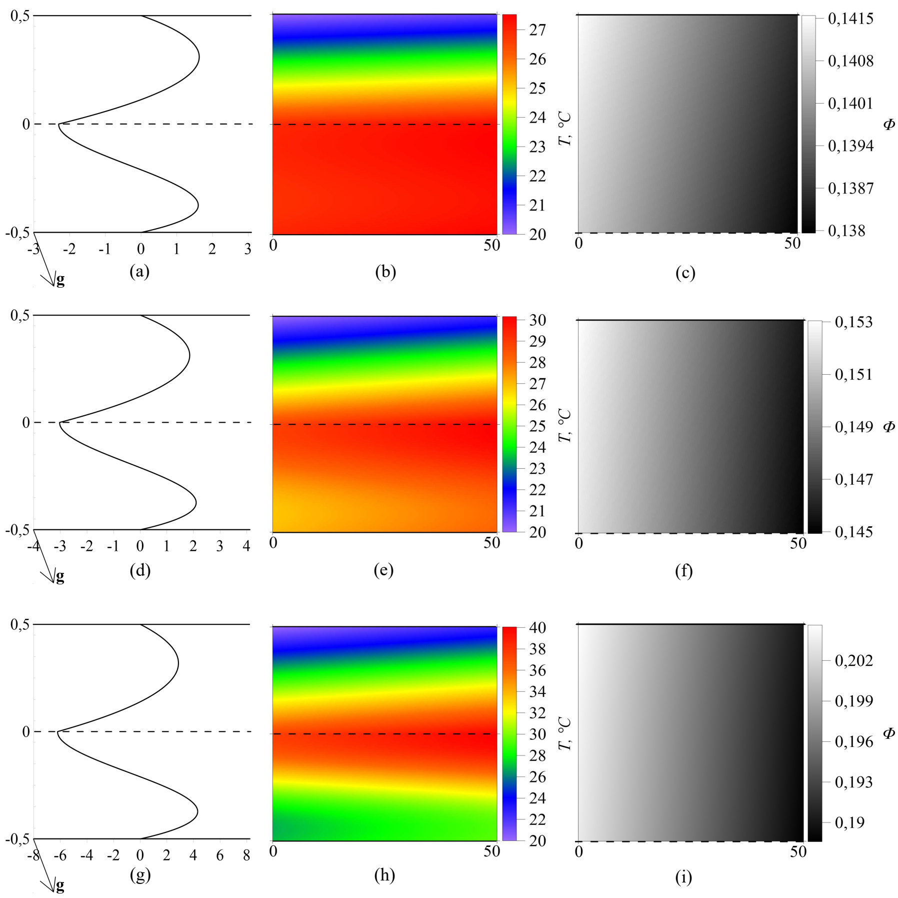

Fig. 4. Velocity profiles (a,d,g), temperature (b,e,h), and vapor concentration (c) distributions in the system: $\vartheta^- = 27\ °C$, $\vartheta^+ = 20\ °C$, $g = 9.81$ m/s^2, (a,b,c) — $A = 1$ K/m, $M = 0.212 \cdot 10^{-6}$ kg/(m^2·c), (d,e,f) — $A = 2.5$ K/m, $M = 0.531 \cdot 10^{-6}$ kg/(m^2·s), (g,h) — $A = 5$ K/m, $M = 1.062 \cdot 10^{-6}$ kg/(m^2·s))

In the case when longitudinal temperature gradients have negative values (the heater is located on the right side, see Fig. 5), some qualitative changes in the velocity profile appear. The intensity of return currents near the interface decreases with increasing modulus of A values. The quantitative characteristics of the temperature distribution change weakly, but the character of the function itself changes. The highest temperature values are conserved near the lower wall of the channel for all A, but the heater influence increases. The picture of the vapor concentration distribution also changes qualitatively with respect to the case when $A > 0$. As the value of $|A|$ increases, the drop in the value of the function Φ also increases. Note, that in this case there is a process of condensation of liquid ($M < 0$).

Conclusions

The work presents exact solutions of a special type of Navier–Stokes equations in the Boussinesq approximation. Mathematical modeling of stationary bilayer convective flows taking into account heat and mass transfer at a non-deformable thermocapillary interface is carried out.

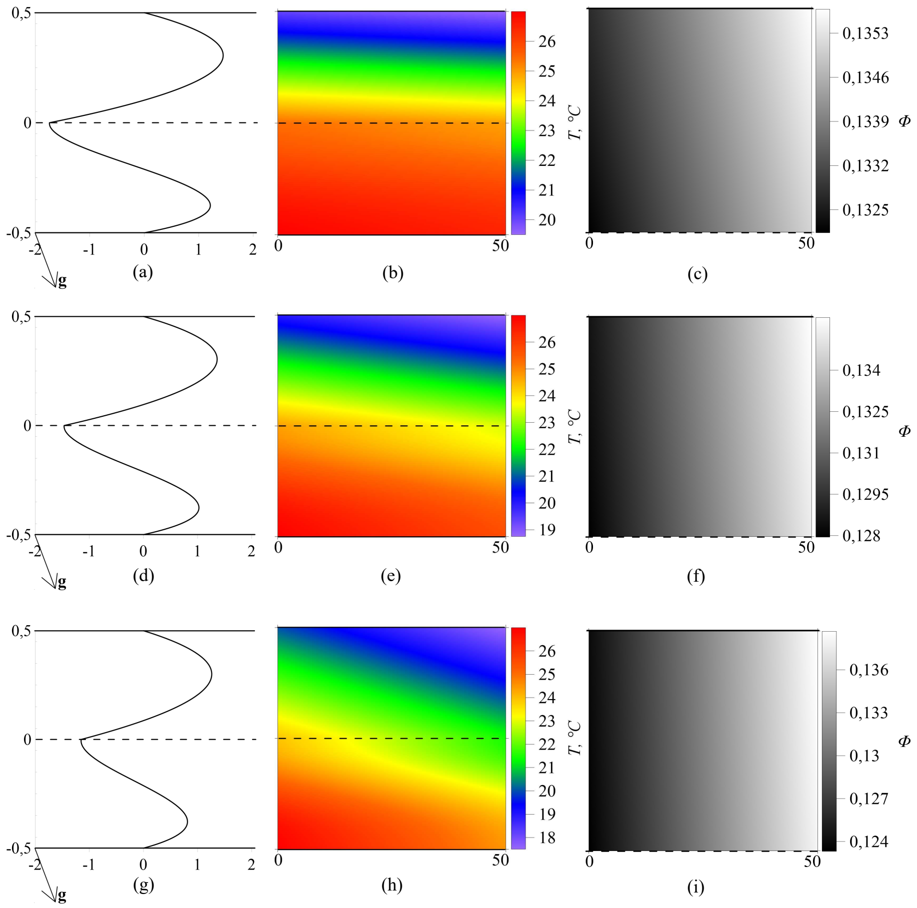

Fig. 5. Velocity profiles (a,d,g), temperature (b,e,h), and vapor concentration (c) distributions in the system: $\vartheta^- = 27\ °C$, $\vartheta^+ = 20\ °C$, $g = 9.81$ m/s^2, (a,b,c) — $A = -1$ K/m, $M = -0.212\cdot 10^{-6}$ kg/(m^2·c), (d,e,f) — $A = -2.5$ K/m, $M = -0.531\cdot 10^{-6}$ kg/(m^2·c), (g,h) — $A = -5$ K/m, $M = -1.062\cdot 10^{-6}$ kg/(m^2·c)

The effects of thermodiffusion and diffusive thermal conductivity are considered in the gas-vapor layer. The condition of zero vapor flux is set on the upper wall of the channel. The flows are modeled for both inclined and horizontal channels. It is shown that in the case of an inclined layer the location of the heater on the system boundaries (the sign of the longitudinal temperature gradient) affects the type of the exact solution.

Examples of longitudinal velocity profiles, distributions of temperature and vapor concentration for the «ethanol — nitrogen» system are given. Comparison of the results obtained by means of exact solutions describing the flow in inclined and horizontal layers is presented. The impact of the channel inclination angle, gravitation level and thermal load on the character of the flows has been studied. It is shown that in the case of normal gravity, an increase in the inclination angle of the system significantly changes the character of the flow, while in conditions of microgravity such effect is not observed.

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Влияние угла наклона и тепловой нагрузки на характер течения в двухслойной системе с учетом массопереноса

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Аннотация. Изучаются двухслойные конвективные течения жидкости и парогазовой смеси в наклонном канале с учетом тепло- и массопереноса на термокапиллярной границе раздела. Математическое моделирование проводится на основе точных решений специального вида уравнений Навье–Стокса в приближении Обербека-Буссинеска с учетом эффектов Соре и Дюфура в газопаровом слое. Наклонное или горизонтальное положение канала, а также направление граничной тепловой нагрузки определяют вид точного решения и алгоритм его построения. Приведены примеры профилей скорости, распределения температуры и концентрации пара в системе «этанол — азот». Представлены результаты сравнительного анализа двухслойных течений в системе, расположенной горизонтально и под небольшим наклоном относительно горизонтального положения. Изучено влияние интенсивности тепловой нагрузки на характер течения и массоперенос на границе раздела.

Ключевые слова: точное решение, двухслойное течение, термокапиллярная граница раздела, конвекция, массоперенос, наклонный канал.